Dynamical Analysis of Predator-Prey Model Leslie-Gower with Omnivore

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Abstract
This article discussed a dynamical analysis on a model of predator-prey Leslie-Gower with omnivores which is modified by Lotka-Volterra model with omnivore. The dynamical analysis was done by determining the equilibrium point with its existing condition and analyzing the local stability of the equilibrium point. Based on the analysis, there are seven points of equilibrium. Three of them always exist while the four others exist under certain conditions. Four points of equilibrium, which are unstable, while the others three equilibrium point are local asymptotically stable under certain conditions. Moreover, numerical simulations were also conducted to illustrate the analytics. The results of numerical simulations agree with the results of the dynamical analysis.

Keywords: local stability, omnivore, predator-prey models, the equilibrium point.

INTRODUCTION

Lotka-Volterra model was firstly introduced Lotka in 1925 and Volterra in 1926 [1]. Lotka-Volterra's study has produced a simple model of predation or interaction between two species in an ecosystem. They also have introduced classical Lotka-Volterra model, which is currently developed by researchers [2].

In 1948, Leslie discussed Lotka-Volterra model and found impossibility in a model, which is infinity in predator growth [3]. Therefore, Leslie and Gower introduced a new name's predator-prey model, which is modification of Lotka-Volterra's model. The model is known as Leslie-Gower Predator-Prey Model. Leslie-Gower have modified Lotka-Volterra's *predator-prey* model by assuming that the predation of predator is limited, which means that the predation of predator will not more carrying capacity of prey. Leslie-Gower two dimension models as:

$$\frac{dx}{dt} = (r_1 - a_1 y - b_1 x)x,$$
$$\frac{dy}{dt} = \left(r_2 - a_2 \frac{y}{x}\right)y. \qquad \textbf{(1)}$$

with $x(t)$ state the population density of prey and $y(t)$ state the population density of predator.

In 2015, Andayani and Kusumawinahyu [4] a three species predator – prey model, the third species are omnivores. This model is constructed by assumming there are just three species in such an ecosystem. The first species, called as prey (rice plant), the prey for the second and the third species. The second species, called as predator (carrion), only feeds on the first species and can extinct with prey. The third species, namely omnivores (mouse), eat the prey and carcasses of predator. Consequently, omnivores of predator only reduces the prey population but does not affect the predator growth. Assumed that the prey population grow logistically and any competition between omnivores [5]. Based on these assumption, the mathematical model representing those growth density of population rates by nonlinear ordinary differential equation system, namely

$$\frac{dx}{dt} = x(1 - y - z - bx),$$
$$\frac{dy}{dt} = y(-c + x) \qquad \textbf{(2)}$$
$$\frac{dz}{dt} = z(-e + fx + gy - \beta z).$$

In this model, $x(t), y(t)$ and $z(t)$ the density of prey, predator, and omnivore populations, respectively. All parameter of model (2) are positive. The death rates of the predator and omnivore are denoted by c and e, respectively. The parameter f rivalry toward prey that effect increases of omnivore population, while the parameter g rivalry toward prey that effect increases of omnivore population. Parameter b and β carrying the capacity of the prey and omnivore, respectively [5]. The aim of this study is a dynamical analysis on a model of predator-prey

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Leslie-Gower with omnivores which is modified by Lotka-Volterra model with omnivore.

MATERIAL AND METHOD

In this study, predator-prey model by Leslie-Gower with omnivore. This model is constructed by assumming the third species are omnivores. This model is constructed by assumming there are just three species in such an ecosystem. The first species, called as prey (rice plant), the prey for the second and the third species. The second species, called as predator (carrion), only feeds on the first species and can extinct with prey. The third species, namely omnivores (mouse), eat the prey and carcasses of predator. Consequently, omnivores of predator only reduces the prey population but does not affect the predator growth. Assumed that the prey population grow logistically and any competition between omnivores.

Literature Study

Literature study related to the research process, such as the literature discussing the Leslie-Gower model, Lotka-Volterra model, omnivore, and forward-backward sweep method [7-12]. We also used other supporting references in problem solving in this study. In the Lotka-Volterra predator-prey model with omnivore, namely:

$$\frac{dx}{dt} = x(1 - y - z - bx),$$
$$\frac{dy}{dt} = y(-c + x),$$
$$\frac{dz}{dt} = z(-e + fx + gy - \beta z).$$

on this system has only five equilibrium point's, namely:

$$E_0 = (0,0,0)$$
$$E_1 = \left(\frac{1}{b}, 0, 0\right)$$
$$E_2 = (c, 1 - bc, 0)$$
$$E_3 = \left(\frac{e+\beta}{f+\beta b}, 0, \frac{f - be}{f - \beta b}\right)$$
$$E_4 = \left(c, \frac{\beta(1-bc) - (fc - e)}{g+\beta}, \frac{g(1-bc) + f(fc-e)}{g+\beta}\right)$$

To accommodate biological meaning, the existence conditions for the equilibria require that they are nonnegative. It is obvious that E_0 dan E_1 always exist, E_2 exist if $bc < 1$, E_3 exist if $f > be$ and E_4 the densities of omnivores and predators $1 - bc$ has to be positive. Then, E_4 exist if $bc < 1$ and $0 < fc - e < \beta(1 - bc)$.

While, predator-prey model by Leslie-Gower with omnivore has seven equilibrium points (Table 1). So the predator-prey model by Leslie-Gower model with omnivores is more concrete in this case.

Table 1. Equilibrium Points of predator-prey model by Leslie-Gower with omnivore

Equilibrium Points	Existence Requirement
$E_1 = (0,0,0)$	-
$E_2 = \left(0, \frac{kr_2}{a_2}, 0\right)$	-
$E_3 = \left(\frac{r_1}{b_1}, 0, 0\right)$	-
$E_4 = (0, y_4, z_4)$	$c_3kr_2 > a_2c_1$
$E_5 = (x_5, 0, z_5)$	$r_1c_2 > b_1c_1$
$E_6 = (x_6, y_6, 0)$	$a_2r_1 > b_2kr_2$
$E_7 = (x_7, y_7, z_7)$	$c_4r_1 + c_1b_3 > \frac{c_1}{c_3}(b_2c_4 + b_3c_3)$

MATHEMATICAL MODEL

This study constructs Lotka-Volterra's *predator-prey* model with omnivore (2). This model is developed by modify the predator that previously used Lotka-Volterra's form to Leslie-Gower's, which was examined by Leslie-Gower. This is based on the fact that predator depends on the available number of prey to establish. Therefore, the model is stated to be in the following equation system (3):

$$\begin{aligned} \frac{dx}{dt} &= x(r_1 - b_1x - b_2y - b_3z), \\ \frac{dy}{dt} &= y\left(r_2 - \frac{a_2}{x+k}y\right), \\ \frac{dz}{dt} &= z(-c_1 + c_2x + c_3y - c_4z) \end{aligned} \tag{3}$$

with $x = x(t)$, $y = y(t)$, and $z = z(t)$ state the population density of prey, predator, and omnivore. All of the parameters are positive in value. Parameters r_1 and r_2 respectively show intrinsic growth of prey and predator. b_1 is the coeffcient of competition between prey, b_2 is the predator interaction coefficient between predator to prey and b_3 is a predatory interaction between omnivores against prey. Whereas a_2 is an interaction parameter between predators and parameter k is a parameter of protection against

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predators. Parameter c_1 is omnivorous natural death, c_2 partially omnivorous predictor coefficient to prey, c_3 as predator coefficient of predator carcass, c_4 is competition between omnivorous population. While, tribal form $\frac{(a_2 y)}{(x+k)}$ can be interpreted as scarcity of prey may stimulate predators to replace foot sources with other alternatives. Therefore, it is assumed that predators depend not only on prey, but predators can eat other than prey in the prey environment. So in this article it is modeled by adding a positive constant k to the division.

RESULT AND DISCUSSION

All parameters of Equation (3.1) in this study are assumed positive in value. Parameters r_1 and r_2 consecutively show intrinsic growth of prey and predator. b_1 is the competition coefficient among preys, b_2 is the predation interaction co-efficient between predator and prey, and b_3 is the predation interaction between omnivore and prey. Meanwhile, a_2 is the interaction parameter among predators, and parameter k is the protec-tion parameter against predator. Parameter c_1 is the natural death of omnivore, c_2 is the preda-tion coefficient of omnivore on prey, c_3 is the predation coefficient on the carcass of predator, and c_4 is the competition among omnivore popu-lations.

Equilibrium Point and Existence

The point of Equilibrium (3) is solution for sytem:

$$x(r_1 - b_1 x - b_2 y - b_3 z) = 0,$$
$$y\left(r_2 - \frac{a_2}{x+k} y\right) = 0,$$
$$z(-c_1 + c_2 x + c_3 y - c_4 z) = 0.$$

The system has seven points of equilibrium, namely $E_1 = (0,0,0)$, $E_2 = \left(0, \frac{kr_2}{a_2}, 0\right)$, and $E_3 = \left(\frac{r_1}{b_1}, 0,0\right)$ in which the three points exist unconditionally, equilibrium point $E_4 = \left(0, \frac{r_2 k}{a_2}, \frac{c_3 r_2 k - c_1 a_2}{c_4 a_2}\right)$ exists with the condi-tion of $c_3 r_2 k > a_2 c_1$, equilibrium point $E_5 = \left(\frac{r_1 c_4 + b_3 c_1}{c_2 b_3 + c_4 b_1}, 0, \frac{c_2 r_1 - c_1 b_1}{c_2 b_3 + c_4 b_1}\right)$ exists if $z = c_2 r_1 > c_1 b_1$, equilibrium point $E_6 = \left(\frac{r_1 a_2 - b_2 r_2 k}{a_2 b_1 + b_2 r_2}, \frac{r_2 (r_1 + b_1 k)}{a_2 b_1 + b_2 r_2}, 0\right)$ exists if $a_2 r_1 > b_2 k r_2$. Equilibrium point $E_7 = (x_7, y_7, z_7)$ with

$$x_7 = \frac{a_2 r_1 c_4 + a_2 c_1 b_3 - r_2 k b_2 c_4 - r_2 k c_3 b_3}{a_2 b_1 c_4 + a_2 c_2 b_3 + r_2 b_2 c_4 + r_2 c_3 b_3}$$
$$y_7 = \frac{r_2 (r_1 c_4 + c_1 b_3 + k b_1 c_4 + k c_2 b_3)}{a_2 b_1 c_4 + a_2 c_2 b_3 + r_2 b_2 c_4 + r_2 c_3 b_3}$$
$$z_7 = \frac{r_1 r_2 c_3 + r_1 a_2 c_2 + b_1 r_2 k c_3 - b_1 a_2 c_1 - b_2 r_2 c_1 - b_2 r_2 k c_2}{r_2 b_2 c_4 + r_2 c_3 b_3 + a_2 b_1 c_4 + a_2 c_2 b_3}$$

exists if $r_1 c_4 + b_3 c_1 > \frac{c_1}{c_3}(b_2 c_4 + b_3 c_3)$.

Stability Analysis

The local stability of system (3) for each equilibrium point is as follows.

a) $E_1 = (0,0,0)$ is unstable
b) $E_2 = \left(0, \frac{r_2 k}{a_2}, 0\right)$ is stable if $r_1 a_2 < b_2 r_2 k$ and $c_3 r_2 k > c_1 a_2$
c) $E_3 = \left(\frac{r_1}{b_1}, 0,0\right)$ is unstable
d) $E_4 = \left(0, \frac{r_2 k}{a_2}, \frac{c_3 r_2 k - c_1 a_2}{c_4 a_2}\right)$ is stable if $c_4 r_1 + c_1 b_3 < \frac{k r_2}{a_2}(b_2 c_4 + b_3 c_3)$
e) $E_5 = \left(\frac{r_1 c_4 + b_3 c_1}{c_2 b_3 + c_4 b_1}, 0, \frac{c_2 r_1 - c_1 b_1}{c_2 b_3 + c_4 b_1}\right)$ is unstable
f) $E_6 = \left(\frac{r_1 a_2 - b_2 r_2 k}{a_2 b_1 + b_2 r_2}, \frac{r_2 (r_1 + b_1 k)}{a_2 b_1 + b_2 r_2}, 0\right)$ is unstable
g) $E_7 = (x_7, y_7, z_7)$.

This stability analysis uses the criteria of Routh-Hurwitz $\lambda^3 + A\lambda^2 + B\lambda + C = 0$. The characteristic equation will have negative roots if and only if $A > 0, C > 0$ and $AB > C$. Therefore, it can be concluded that $AB - C > 0$. If the condition is met, the equi-librium point of E_7 will be stable.

Proof

i. Jacobi matrix system (3) for E_1 is $J(E_1) = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & -c_1 \end{bmatrix}$. The three Eigen values of the matrix $J(E_1)$ are positive, so E_1 is unstable.

ii. Jacobi matrix $E_2 = \left(0, \frac{r_2 k}{a_2}, 0\right)$ is

$$J = \begin{bmatrix} r_1 - \frac{b_2 r_2 k}{a_2} & 0 & 0 \\ \frac{r_2{}^2}{a_2} & -r_2 & 0 \\ 0 & 0 & -c_1 + \frac{c_3 r_2 k}{a_2} \end{bmatrix},$$

which has eigen values $\lambda_1 = r_1 - \frac{b_2 r_2 k}{a_2}$, $\lambda_2 = -r_2$, $\lambda_3 = -c_1 + \frac{c_3 r_2 k}{a_2}$

. The equilibrium point (E_2) stable, if $r_1 a_2 < b_2 r_2 k$ and $c_3 r_2 k > c_1 a_2$.

iii. Jacobi matrix on equilibrium point $E_3 = \left(\frac{r_1}{b_1}, 0, 0\right)$ is

$$J = \begin{bmatrix} -r_1 & -\dfrac{r_1 b_2}{b_1} & -\dfrac{r_1 b_3}{b_1} \\ 0 & r_2 & 0 \\ 0 & 0 & -c_1 + \dfrac{c_2 r_1}{b_1} \end{bmatrix},$$

has eigen values $\lambda_1 = -r_1,\ \lambda_2 = r_2,\ \lambda_3 = -c_1 + \frac{c_3 r_1}{b_1}$ so E_3 unstable.

iv. Jacobi matrix on equilibrium point $E_4 = \left(0, \frac{r_2 k}{a_2}, \frac{c_3 r_2 k - c_1 a_2}{c_4 a_2}\right)$ is

$$J = \begin{bmatrix} r_1 - b_2 y_4 - b_3 z_4 & 0 & 0 \\ \dfrac{r_2^{\,2}}{a_2} & -r_2 & 0 \\ c_2 z_4 & c_3 z_4 & -c_1 - 2c_4 z_4 + c_3 z_4 \end{bmatrix},$$

because $x = 0, r_2 \frac{a_2}{x+k} y = 0$, and $-c_1 + c_2 x + c_3 y - c_4 z = 0$. so E_4 stable if

$$\lambda_1 = r_1 - \frac{b_2 k r_2}{a_2} - \frac{b_3}{a_2 c_4}(k r_2 c_3 - a_2 c_1) < 0,$$

$$\frac{r_1 a_2 c_4 - b_2 k r_2 c_4 - b_3 k r_2 c_3 + b_3 a_2 c_1}{a_2 c_4} < 0,$$

$$c_4 r_1 + c_1 b_3 < \frac{k r_2}{a_2}(b_2 c_4 + b_3 c_3).$$

v. Jacobi matrix on the equilibrium point $E_5 = \left(\frac{r_1 c_4 + b_3 c_1}{c_2 b_3 + c_4 b_1},\ 0,\ \frac{c_2 r_1 - c_1 b_1}{c_2 b_3 + c_4 b_1}\right)$

$$J(E_5) = \begin{bmatrix} r_1 - 2b_1 x_5 - b_3 z_5 & -b_2 x & -b_3 x \\ 0 & r_2 & 0 \\ c_2 z_5 & c_3 z_5 & -c_1 - 2c_4 z_5 + c_2 x_5 \end{bmatrix}$$

with,

$$r_1 - b_1 x - b_2 y - b_3 z = 0 \Rightarrow r_1 - b_1 x_5 - b_3 z_5 = 0,$$
$$y = 0,$$
$$-c_1 + c_2 x + c_3 y - c_4 z = 0 \Rightarrow -c_1 + c_2 x_5 - c_4 z_5 = 0,$$

so

$$J(E_5) = \begin{bmatrix} -b_1 x_5 & -b_2 x & -b_3 x \\ 0 & r_2 & 0 \\ c_2 z_5 & c_3 z_5 & -c_4 z_5 \end{bmatrix},$$

$$|J(E_5) - \lambda I| = \begin{vmatrix} -b_1 x_5 - \lambda & -b_2 x & -b_3 x \\ 0 & r_2 - \lambda & 0 \\ c_2 z_5 & c_3 z_5 & -c_4 z_5 - \lambda \end{vmatrix},$$

$$= (r_2 - \lambda_1)\begin{vmatrix} -b_1 x_5 - \lambda & -b_3 x \\ c_2 z_5 & -c_4 z_5 - \lambda \end{vmatrix} = 0.$$

The equilibrium point E_5 result $\lambda_1 = r_2 > 0$ so unstable l.

vi. Jacobi matrix on the equilibrium point $E_6 = \left(\frac{r_1 a_2 - b_2 r_2 k}{a_2 b_1 + b_2 r_2}, \frac{r_2 (r_1 + b_1 k)}{a_2 b_1 + b_2 r_2}, 0\right)$

$$J(E_6) = \begin{bmatrix} r_1 - 2b_1 x_6 - b_2 y_6 & -b_2 x_6 & -b_3 x_6 \\ \dfrac{a_2 y^2}{(x+k)^2} & r_2 \dfrac{2a_2 y}{x+k} & 0 \\ 0 & 0 & -c_1 + c_2 x_6 + c_3 y_6 \end{bmatrix},$$

with,

$$r_1 - b_1 x - b_2 y - b_3 z = 0 \Rightarrow r_1 - b_1 x_6 - b_3 y_6 = 0,$$
$$r_2 \frac{a_2}{x+k} y = 0 \Rightarrow \frac{y}{x+k} = \frac{r_2}{a_2} \rightarrow \left(\frac{y}{x+k}\right)^2 = \frac{r_2^{\,2}}{a_2^{\,2}},$$
$$z = 0,$$

so

$$J(E_6) = \begin{bmatrix} -b_1 x_6 & -b_2 x & -b_3 x \\ \dfrac{r_2^{\,2}}{a_2} & -\dfrac{a_2 y}{x+k} & 0 \\ 0 & 0 & -c_1 + c_2 x_6 + c_3 y \end{bmatrix},$$

$$|J(E_6) - \lambda I| = \begin{vmatrix} -b_1 x_6 - \lambda & -b_2 x & -b_3 x \\ \dfrac{r_2^{\,2}}{a_2} & -\dfrac{a_2 y}{x+k} - \lambda & 0 \\ 0 & 0 & J_{33} - \lambda \end{vmatrix},$$

$$= (J_{33} - \lambda)\left[(\lambda + b_1 x)\left(\lambda + \frac{a_2 y}{x+k}\right) + \frac{b_2 r_2^{\,2}}{a_2} x\right] = 0,$$

$$= (J_{33} - \lambda)\left[\lambda^2 + \left(b_1 x + \frac{a_2 y}{x+k}\right)\lambda + \frac{b_1 a_2 y}{x+k} + \frac{b_2 r_2^{\,2}}{a_2} x\right] = 0.$$

result $\lambda_1 < 0$, with used characeristic as follow $\lambda_1 \lambda_2 > 0 \wedge \lambda_1 + \lambda_2 < 0$ proven that E_6 unstable.

vii. Jacobi matrix on equilibrium point $E_7 = (x_7, y_7, z_7)$

$$J(E_7) = \begin{bmatrix} b_1 x_7 & -b_2 x_7 & -b_3 x_7 \\ \dfrac{a_2 y^2}{(x+k)^2} & r_2 \dfrac{2a_2 y}{x+k} & 0 \\ c_2 z_7 & c_3 z_7 & -c_1 z_7 \end{bmatrix},$$

$$J(E_7) = \begin{bmatrix} -b_1 x_7 & -b_2 x & -b_3 x \\ \dfrac{r_2^{\,2}}{a_2} & -r_2 & 0 \\ c_2 z_7 & c_3 z_7 & -c_1 z_7 \end{bmatrix},$$

Same of characteristic with use cofactor expansion second line,

$$|J(E_7) - \lambda I| = \begin{vmatrix} -b_1 x_7 - \lambda & -b_2 x_7 & -b_3 x_7 \\ \dfrac{r_2^{\,2}}{a_2} & -r_2 - \lambda & 0 \\ c_2 z_7 & c_3 z_7 & -c_1 z_7 - \lambda \end{vmatrix},$$

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$$= -\frac{{r_2}^2}{a_2}\begin{vmatrix} -b_2x_7 & -b_3x_7 \\ c_3z_7 & -c_1z_7-\lambda \end{vmatrix} + (-r_2-\lambda)\begin{vmatrix} -b_1x_7-\lambda & -b_3x_7 \\ c_2z_7 & -c_1z_7-\lambda \end{vmatrix} = 0,$$

$$= -\frac{1}{a_2}(a_2b_1c_4\lambda x_7z_7 + a_2b_1c_4r_2x_7z_7 + a_2b_3c_2\lambda x_7z_7 + a_2b_3c_2r_2x_7z_7$$
$$+b_2c_4{r_2}^2x_7z_7 + b_3c_3{r_2}^2x_7z_7 + a_2b_1\lambda^2x_7 + a_2b_1\lambda r_2x_8 + a_2c_4\lambda^2z_7$$
$$+a_2c_4\lambda r_2z_7 + b_2\lambda {r_2}^2x_7 + a_2\lambda^3 + a_2\lambda^2r_2),$$

With declaration as:

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0,$$

with,

$$A = c_4z_7 + b_1x_7 + r_2 > 0,$$
$$B = b_1c_4x_7z_7 + b_3c_2x_7z_7 + b_1r_2x_8 + c_4r_2z_7 + a_2{r_2}^2x_7 > 0,$$
$$C = \frac{a_2b_1c_4r_2x_7z_7 + a_2b_3c_2r_2x_7z_7 + b_2c_4{r_2}^2x_7z_7 + b_3c_3{r_2}^2x_7z_7}{a_2}.$$

Based on criteria Routh-Hurwitz, the equilibrium point E_7 local asymptotically stable if $A > 0,\ C > 0$ and $AB - C > 0$.

Numerical Simulation

Numerical simulations are performed to see the validity of numerical analysis using the fourth-orde Runge-Kutta method to ilustrate the results of the analysis. There are several cases that are simulated in the discussion of this study, as follows.

Simulation I

Simulation I (Fig. 1) show E_2 exists, and conditions of stable E_2 are $r_1a_2 < b_2kr_2$ and $c_1a_2 > c_3kr_2$, Parameter being used $r_1 = 0.5,\ b_1 = 0.05,\ b_2 = 0.6,\ b_3 = 0.1,\ r_2 = 0.7,\ a_2 = 0.2,\ k = 0.3,\ c_1 = 0.9,\ c_2 = 0.25,\ c_3 = 0.2,\ c_4 = 0.2$

Thus, the equilibrium points of E_1 exists, E_2 exist, E_3 exists, and E_5 exists. The numerical simulation to equilibrium point $E_2 = (0{,}1.05{,}0)$. This is relevant to the analysis result which states that equilibrium E_2 is stable.

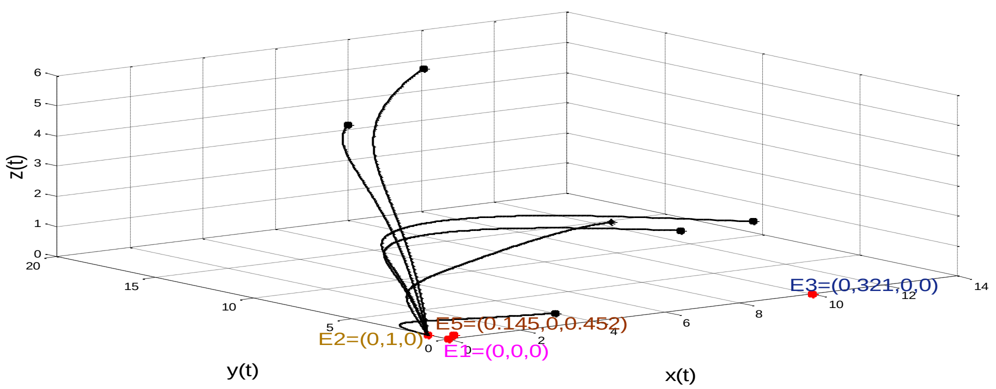

Figure 1. Portrait Phase System Equation 3.1 for simulation I

Simulation II

Simulation II, the stability conditions of E_2 are changed into $r_1a_2 > b_2kr_2$ and $c_1a_2 < c_3kr_2$, parameter $r_1 = 0.8, b_2 = 0.4, b_3 = 0.1, b_1 = 0.05, r_2 = 0.7,\ a_2 = 0.2, k = 0.3, c_1 = 0.3, c_2 = 0.25, c_3 = 0.6, c_4 = 0.2$. Thus, it produces E_4 exists, E_5 exists, E_6 exists. Then, E_7 exists and is stable toward equilibrium point, so the initial value shows that E_7 is stable.

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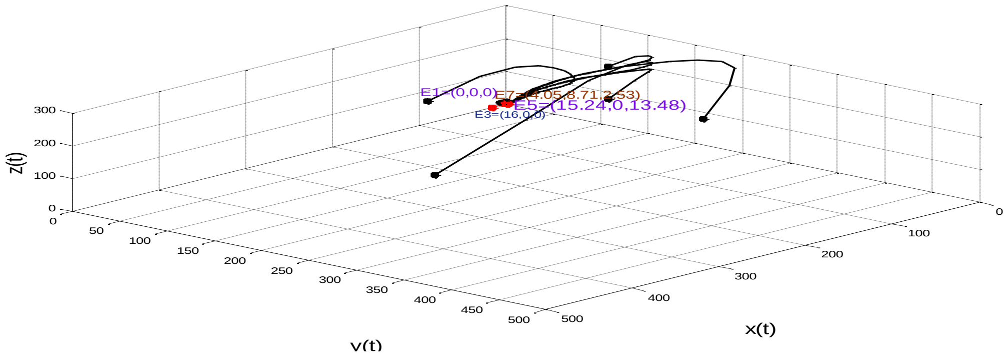

Figure 2. Portrait Phase System Equation 3.1 for Simulation II

Simulation III

The stability conditions in simulation III of E_2 are changed into $r_1 a_2 > b_2 k r_2$ dan $c_1 a_2 > c_3 k r_2$, parameter $r_1 = 0.8, b_2 = 0.4, b_3 = 0.1, b_1 = 0.05, r_2 = 0.7, a_2 = 0.2, k = 0.3, c_1 = 0.9, c_2 = 0.25,$ $c_3 = 0.2, c_4 = 0.2$. Thus, it produces E_1 exists, E_2 exists, E_3 exists, E_5 exists, and E_6 exists, but, in this case, it does not go to any point, so it exists but is unstable.

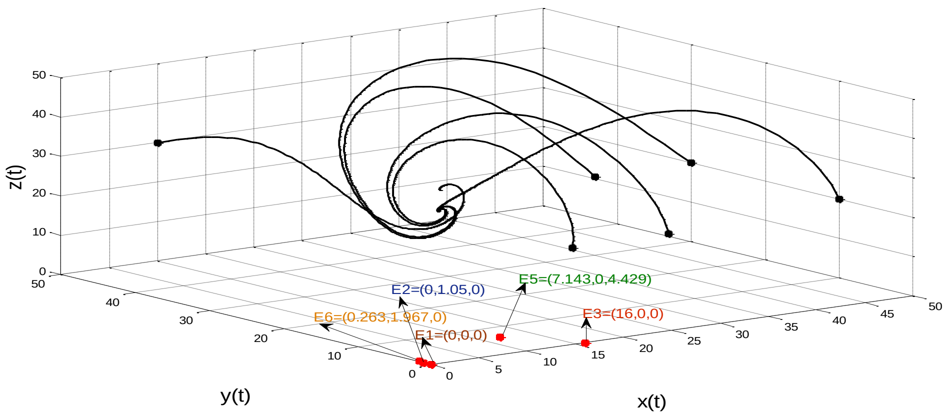

Figure 3. Portrait Phase System Equation 3.1 for Simulation III

Simmulation IV

The stability conditions in simulation IV, of E_2 are changed into $r_1 a_2 < b_2 k r_2$ dan $c_1 a_2 < c_3 k r_2$, parameter $r_1 = 0.5, b_2 = 0.6, b_3 = 0.1, b_1 = 0.05,$ $r_2 = 0.7, a_2 = 0.2, k = 0.3, c_1 = 0.3, c_2 = 0.25,$ $c_3 = 0.6, c_4 = 0.2$. Thus, it produces E_1 exists, E_2 exists, E_3 exists, E_4 exists, and E_5 exists and unstable.

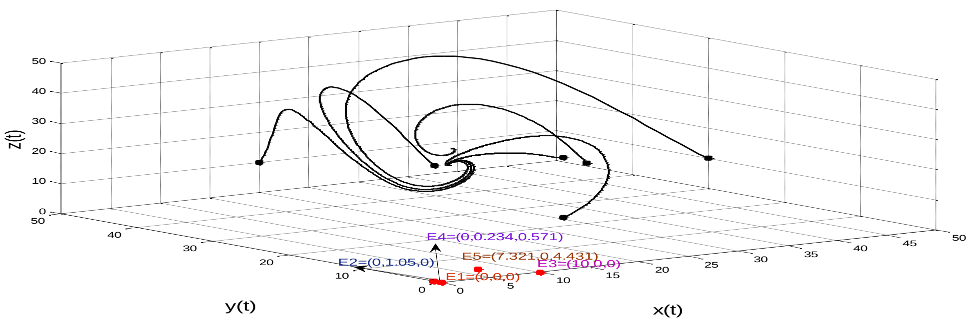

Figure 4. portrait phase system equation 3.1 for simulation IV

Simulation V:

Stability condition $c_4 r_1 + c_1 b_3 < \frac{r_2 k}{a_2}(b_2 c_4 + b_3 c_3)$, the fifth simulation is conducted to show the stability characteristics of equilibrium point E_4. Based on the existence condition and the stability of equilibrium point E_4, parameter $r_1 = 0.5, b_2 = 0.4, b_3 = 0.1, b_1 = 0.05, r_2 = 0.9, a_2 = 0.2, k = 0.5, c_1 = 0.3, c_2 = 0.25, c_3 = 0.6, c_4 = 0.2$. Thus, it produces E_1 exists, E_2 exists, E_3 exists, E_4 exists, and E_5 exists towards equilibrium point E_4. This is relevant to the analysis result which states that the equilibrium point is stable.

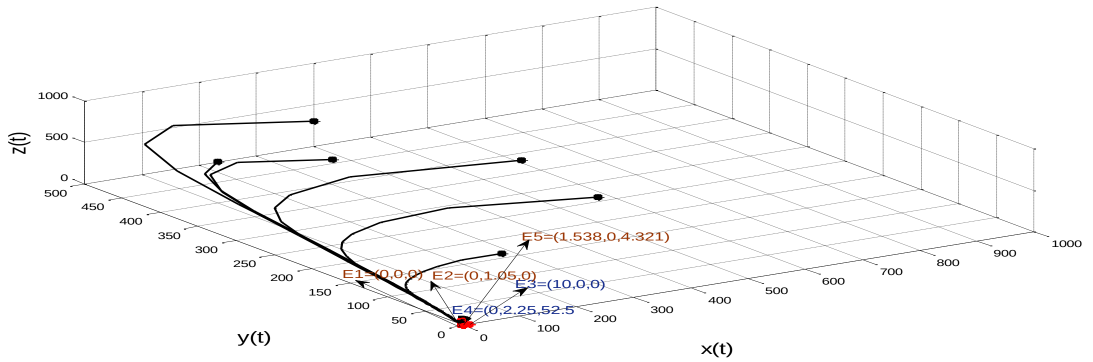

Figure 5. Portrait Phase System Equation 3.1 for Simulation V

CONCLUSION

The conclusions that are drawn based on the discussion of the thesis are as follows. Predator-prey model by Leslie-Gower with omnivore is obtained in a form of common differential equation system. There are seven equilibrium points in the model, there are three of them, i.e. E_1, E_2 and E_3, unconditionally exist and the other four, i.e. E_4, E_5, E_6 and E_7, conditionally exist. Of the seven equilibrium points, three of them, E_1, E_2, E_4 dan E_7, have stability condition.

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